Topos Theory

This book studies the foundations of quantum theory through its relationship to classical physics. This idea goes back to the Copenhagen Interpretation (in the original version due to Bohr and Heisenberg), which the author relates to the mathematical formalism of operator algebras originally created by von Neumann. The book therefore includes comprehensive appendices on functional analysis and C*-algebras, as well as a briefer one on logic, category theory, and topos theory. Matters of foundational as well as mathematical interest that are covered in detail include symmetry (and its "spontaneous" breaking), the measurement problem, the Kochen-Specker, Free Will, and Bell Theorems, the Kadison-Singer conjecture, quantization, indistinguishable particles, the quantum theory of large systems, and quantum logic, the latter in connection with the topos approach to quantum theory. This book is Open Access under a CC BY licence.

This text introduces topos theory, a development in category theory that unites important but seemingly diverse notions from algebraic geometry, set theory, and intuitionistic logic. Topics include local set theories, fundamental properties of toposes, sheaves, local-valued sets, and natural and real numbers in local set theories. 1988 edition.

"It would be hard to imagine a better guide to this difficult subject."--Scientific American

In Three Roads to Quantum Gravity, Lee Smolin provides an accessible overview of the attempts to build a final "theory of everything." He explains in simple terms what scientists are talking about when they say the world is made from exotic entities such as loops, strings, and black holes and tells the fascinating stories behind these discoveries: the rivalries, epiphanies, and intrigues he witnessed firsthand. "Provocative, original, and unsettling." -The New York Review of Books "An excellent writer, a creative thinker."-Nature

The first of its kind, this book presents a widely accessible exposition of topos theory, aimed at the philosopher-logician as well as the mathematician. It is suitable for individual study or use in class at the graduate level (it includes 500 exercises). It begins with a fully motivated introduction to category theory itself, moving always from the particular example to the abstract concept. It then introduces the notion of elementary topos, with a wide range of examples and goes on to develop its theory in depth, and to elicit in detail its relationship to Kripke's intuitionistic semantics, models of classical set theory and the conceptual framework of sheaf theory (``localization'' of truth). Of particular interest is a Dedekind-cuts style construction of number systems in topoi, leading to a model of the intuitionistic continuum in which a ``Dedekind-real'' becomes represented as a ``continuously-variable classical real number''. The second edition contains a new chapter, entitled Logical Geometry, which introduces the reader to the theory of geometric morphisms of Grothendieck topoi, and its model-theoretic

rendering by Makkai and Reyes. The aim of this chapter is to explain why Deligne's theorem about the existence of points of coherent topoi is equivalent to the classical Completeness theorem for ``geometric'' first-order formulae.
A short introduction ideal for students learning category theory for the first time.
According to Grothendieck, the notion of topos is "the bed or deep river where come to be married geometry and algebra, topology and arithmetic, mathematical logic and category theory, the world of the continuous and that of discontinuous or discrete structures". It is what he had "conceived of most broad to perceive with finesse, by the same language rich of geometric resonances, an "essence" which is common to situations most distant from each other, coming from one region or another of the vast universe of mathematical things". The aim of this book is to present a theory and a number of techniques which allow to give substance to Grothendieck's vision by building on the notion of classifying topos educed by categorical logicians.
Mathematical theories (formalized within first-order logic) give rise to geometric objects called sites; the passage from sites to their associated toposes embodies the passage from the logical presentation of theories to their mathematical content, i.e. from syntax to semantics. The essential ambiguity given by the fact that any topos is associated in general with an infinite number of theories or different sites allows to study the relations between different theories, and hence the theories themselves, by using toposes as 'bridges' between these different presentations. The expression or calculation of

invariants of toposes in terms of the theories associated with them or their sites of definition generates a great number of results and notions varying according to the different types of presentation, giving rise to a veritable mathematical morphogenesis. In the last five decades various attempts to formulate theories of quantum gravity have been made, but none has fully succeeded in becoming the quantum theory of gravity. One possible explanation for this failure might be the unresolved fundamental issues in quantum theory as it stands now. Indeed, most approaches to quantum gravity adopt standard quantum theory as their starting point, with the hope that the theory's unresolved issues will get solved along the way. However, these fundamental issues may need to be solved before attempting to define a quantum theory of gravity. The present text adopts this point of view, addressing the following basic questions: What are the main conceptual issues in quantum theory? How can these issues be solved within a new theoretical framework of quantum theory? A possible way to overcome critical issues in present-day quantum physics – such as a priori assumptions about space and time that are not compatible with a theory of quantum gravity, and the impossibility of talking about systems without reference to an external observer – is through a reformulation of quantum theory in terms of a different mathematical framework called topos theory. This course-tested primer sets out to explain to graduate students and newcomers to the field alike, the reasons for choosing topos theory to resolve the above-mentioned issues and how it brings quantum physics back

to looking more like a “neo-realist” classical physics theory again.
The ubiquitous nature of mobile and pervasive computing has begun to reshape and complicate our notions of space, time, and identity. In this collection, over thirty internationally recognized contributors reflect on ubiquitous computing’s implications for the ways in which we interact with our environments, experience time, and develop identities individually and socially. Interviews with working media artists lend further perspectives on these cultural transformations. Drawing on cultural theory, new media art studies, human-computer interaction theory, and software studies, this cutting-edge book critically unpacks the complex ubiquity-effects confronting us every day. The companion website can be found here: http://ubiquity.dk
Intended for category theorists and logicians familiar with basic category theory, this book focuses on categorical model theory, which is concerned with the categories of models of infinitary first order theories, called accessible categories. The starting point is a characterization of accessible categories in terms of concepts familiar from Gabriel-Ulmer's theory of locally presentable categories. Most of the work centers on various constructions (such as weighted bilimits and lax colimits), which, when performed on accessible categories, yield new accessible categories. These constructions are necessarily 2-categorical in nature; the authors cover some aspects of 2-category theory, in addition to some

basic model theory, and some set theory. One of the main tools used in this study is the theory of mixed sketches, which the authors specialize to give concrete results about model theory. Many examples illustrate the extent of applicability of these concepts. In particular, some applications to topos theory are given. Perhaps the book's most significant contribution is the way it sets model theory in categorical terms, opening the door for further work along these lines. Requiring a basic background in category theory, this book will provide readers with an understanding of model theory in categorical terms, familiarity with 2-categorical methods, and a useful tool for studying toposes and other categories.

Category theory is unmatched in its ability to organize and layer abstractions and to find commonalities between structures of all sorts. No longer the exclusive preserve of pure mathematicians, it is now proving itself to be a powerful tool in science, informatics, and industry. By facilitating communication between communities and building rigorous bridges between disparate worlds, applied category theory has the potential to be a major organizing force. This book offers a self-contained tour of applied category theory. Each chapter follows a single thread motivated by a real-world application and discussed with category-theoretic tools. We see data migration as an adjoint functor, electrical circuits in terms of monoidal categories and operads, and collaborative design via enriched profunctors. All the relevant category theory, from simple to sophisticated, is introduced in an accessible way with many examples and exercises, making this an ideal guide even for those without experience of university-level mathematics. According to Grothendieck, the notion of topos is "the bed or deep river where come to be married geometry and algebra, topology and arithmetic, mathematical logic and category theory, the world of the continuous and that of discontinuous or discrete structures." It is what he had "conceived of most broad to perceive with finesse, by the same language rich of geometric resonances, an "essence" which is common to situations most distant from each other, coming from one region or another of the vast universe of mathematical things." The aim of this book is to present a theory and a number of techniques which allow to give substance to Grothendieck's vision by building on the notion of classifying topos educed by categorical logicians. Mathematical theories (formalized within first-order logic) give rise to geometric objects called sites; the passage from sites to their associated toposes embodies the passage from the logical presentation of theories to their mathematical content, i.e. from syntax to semantics. The essential ambiguity given by the fact that any topos is associated in general with an infinite number of theories or different sites allows to study the relations between different theories, and hence the theories themselves, by using toposes as 'bridges' between these different presentations. The expression or calculation of invariants of toposes in terms of the theories associated with them or their sites of definition generates a great number of results and notions varying according to the different types of presentation, giving rise to a veritable mathematical morphogenesis.

With contributions by numerous experts

This book formally introduces synthetic differential topology, a natural extension of the theory of synthetic differential geometry which captures classical concepts of differential geometry and topology by means of the rich categorical structure of a necessarily non-Boolean topos and of the systematic use of logical infinitesimal objects in it. Beginning with an introduction to those parts of topos theory and synthetic differential geometry necessary for the remainder, this clear and comprehensive text covers the general theory of synthetic differential topology and several applications of it to classical mathematics, including the calculus of variations, Mather's theorem, and Morse theory on the classification of singularities. The book represents the state of the art in synthetic differential topology and will be of interest to researchers in topos theory and to mathematicians interested in the categorical foundations of differential geometry

and topology.
This monograph presents a new, systematic treatment of the relation between classifying topoi and classifying spaces of topological categories. Using a new generalized geometric realization which applies to topoi, a weak homotopy equival- ence is constructed between the classifying space and the classifying topos of any small (topological) category. Topos theory is then applied to give an answer to the question of what structures are classified by "classifying" spaces. The monograph should be accessible to anyone with basic knowledge of algebraic topology, sheaf theory, and a little topos theory.
Algebra, Topology, and Category Theory: A Collection of Papers in Honor of Samuel Eilenberg is a collection of papers dealing with algebra, topology, and category theory in honor of Samuel Eilenberg. Topics covered range from large modules over artin algebras to two-dimensional Poincaré duality groups, along with the homology of certain H-spaces as group ring objects. Variable quantities and variable structures in topoi are also discussed. Comprised of 16 chapters, this book begins by looking at the relationship between the representation theories of finitely generated and large (not finitely generated) modules over an artin algebra. The reader is then introduced to reduced bar constructions on deRham complexes; some properties of two-dimensional Poincaré duality

groups; and properties invariant within equivalence types of categories. Subsequent chapters explore the work of Samuel Eilenberg in topology; local complexity of finite semigroups; global dimension of ore extensions; and the spectrum of a ringed topos. This monograph will be a useful resource for students and practitioners of algebra and mathematics.

Focusing on topos theory's integration of geometric and logical ideas into the foundations of mathematics and theoretical computer science, this volume explores internal category theory, topologies and sheaves, geometric morphisms, and other subjects. 1977 edition.

Part I indicates that typed-calculi are a formulation of higher-order logic, and cartesian closed categories are essentially the same. Part II demonstrates that another formulation of higher-order logic is closely related to topos theory.

A Collection of Lectures by Variuos Authors

This comprehensive monograph presents a detailed overview of creative works by the author and other 20th-century logicians that includes applications of proof theory to logic as well as other areas of mathematics. 1975 edition.

In 'Higher Topos Theory', Jacob Lurie presents the foundations of this theory using the language of weak Kan complexes introduced by Boardman and Vogt, and shows how existing theorems in algebraic topology can be reformulated and generalized in the theory's new language.

In this book, first published in 2003, categorical algebra is used to build a foundation for the study of geometry, analysis, and algebra.

One of the most challenging problems of contemporary theoretical physics is the mathematically rigorous construction of a theory which describes gravitation and the other fundamental physical interactions within a common framework. The physical ideas which grew from attempts to develop such a theory require highly advanced mathematical methods and radically new physical concepts. This book presents different approaches to a rigorous unified description of quantum fields and gravity. It contains a carefully selected cross-section of lively discussions which took place in autumn 2010 at the fifth conference "Quantum field theory and gravity - Conceptual and mathematical advances in the search for a unified framework" in Regensburg, Germany. In the tradition of the other proceedings covering this series of conferences, a special feature of this book is the exposition of a wide variety of approaches, with the intention to facilitate a comparison. The book is mainly addressed to mathematicians and physicists who are interested in fundamental questions of mathematical physics. It allows the reader to obtain a broad and up-to-date overview of a fascinating active research area.

As its title suggests, this book is an introduction to three ideas and the connections between them. Before describing the content of the book in detail, we describe each concept briefly. More extensive introductory descriptions of each concept are in the introductions and notes to Chapters 2, 3 and 4. A topos is a special kind of category defined by axioms saying roughly that certain constructions one can make with sets can be done in the category. In that sense, a topos is a generalized set theory. However, it originated with Grothendieck and Giraud as an abstraction of the of the category of sheaves of sets on a topological space. Later, properties Lawvere and Tierney introduced a more general id~a which they called "elementary topos" (because their axioms did not quantify over sets), and they and other mathematicians developed the idea that a theory in the sense of mathematical logic can be regarded as a topos, perhaps after a process of completion. The concept of triple originated (under the name "standard construc in Godement's book on sheaf theory for the purpose of computing tions") sheaf cohomology. Then Peter Huber discovered that triples capture much of the information of adjoint pairs. Later Linton discovered that triples gave an equivalent approach to Lawverc's theory of equational theories (or rather the infinite generalizations of that theory). Finally, triples have turned out to be a very important tool for deriving various properties of toposes.

The book covers elementary aspects of category theory and topos theory. It has few mathematical prerequisites, and uses categorical methods throughout rather than beginning with set theoretic foundations. It works with key notions such as cartesian closedness, adjunctions, regular categories, and the internal logic of a topos. Full statements and elementary proofs are given for the central theorems, including the fundamental theorem of toposes, the sheafification theorem, and the construction of Grothendieck toposes over any topos as base. Three chapters discuss applications of toposes in detail, namely to sets, to basic differential geometry, and to recursive analysis. - ;Introduction; PART I: CATEGORIES: Rudimentary structures in a category; Products, equalizers, and their duals; Groups; Sub-objects, pullbacks, and limits; Relations; Cartesian closed categories; Product operators and others; PART II: THE CATEGORY OF CATEGORIES: Functors and categories; Natural transformations; Adjunctions; Slice categories; Mathematical foundations; PART III: TOPOSES: Basics; The internal language; A soundness proof for topos logic; From the internal language to the topos; The fundamental theorem; External semantics; Natural number objects; Categories in a topos; Topologies; PART IV: SOME TOPOSES: Sets; Synthetic differential geometry; The effective topos; Relations in regular categories; Further reading; Bibliography; Index. -

Sheaves arose in geometry as coefficients for cohomology and as descriptions of the functions appropriate to various kinds of manifolds. Sheaves also appear in logic as carriers for models of set theory. This text presents topos theory as it has developed from the study of sheaves. Beginning with several examples, it explains the underlying ideas of topology and sheaf theory as well as the general theory of elementary toposes and geometric morphisms and their relation to logic.

This advanced course, a sequel to the first volume of this lecture series on topos quantum theory, delves deeper into the theory, addressing further technical aspects and recent advances. These include, but are not limited to, the development of physical quantities and self-adjoint operators; insights into the quantization process; the description of an alternative, covariant version of topos quantum theory; and last but not least, the development of a new concept of spacetime. The book builds on the concepts introduced in the first volume (published as Lect. Notes Phys. 868), which presents the main building blocks of the theory and how it could provide solutions to interpretational problems in quantum theory, such as: What are the main conceptual issues in quantum theory? And how can these issues be solved within a new theoretical framework of quantum theory? These two volumes together provide a complete, basic course on topos quantum theory, offering a set of mathematical tools to readers interested in tackling fundamental issues in quantum theory in general, and in quantum gravity in particular. From the reviews of the first volume: The book is self-contained and can be used as a textbook or self-study manual teaching the usage of category theory and topos theory, in particular in theoretical physics or in investigating the foundations of quantum theory in mathematically rigorous terms. [The] book is a very welcome contribution. Frank Antonsen, Mathematical Reviews, December, 2013

This is the first volume of the second edition of the now classic book “The Topos of Music”. The author explains the theory's conceptual framework of denotators and forms, the classification of local and global musical objects, the mathematical models of harmony and counterpoint, and topologies for rhythm and motives.

Please note that the content of this book primarily consists of articles available from Wikipedia or other free sources online. Pages: 38. Chapters: Effective topos, Grothendieck topology,

History of topos theory, Nisnevich topology, Subobject classifier.
This book is an introduction to a functorial model theory based on infinitary language categories. The author introduces the properties and foundation of these categories before developing a model theory for functors starting with a countable fragment of an infinitary language. He also presents a new technique for generating generic models with categories by inventing infinite language categories and functorial model theory. In addition, the book covers string models, limit models, and functorial models.
Examining archival material and post-war scholarly and popular literature, Kranjc describes the often sharp divide between Communist-era interpretations of collaboration and those of their émigré anti-Communist opponents.
This innovative monograph explores a new mathematical formalism in higher-order temporal logic for proving properties about the behavior of systems. Developed by the authors, the goal of this novel approach is to explain what occurs when multiple, distinct system components interact by using a category-theoretic description of behavior types based on sheaves. The authors demonstrate how to analyze the behaviors of elements in continuous and discrete dynamical systems so that each can be translated and compared to one another. Their temporal logic is also flexible enough that it can serve as a framework for other logics that work with similar models. The book begins with a discussion of behavior types, interval domains, and translation invariance, which serves as the groundwork for temporal type theory. From there, the authors lay out the logical preliminaries they need for their temporal modalities and explain the soundness of those logical semantics. These results are then applied to hybrid dynamical systems, differential equations, and labeled transition systems. A case study

involving aircraft separation within the National Airspace System is provided to illustrate temporal type theory in action. Researchers in computer science, logic, and mathematics interested in topos-theoretic and category-theory-friendly approaches to system behavior will find this monograph to be an important resource. It can also serve as a supplemental text for a specialized graduate topics course.

Introduction to concepts of category theory — categories, functors, natural transformations, the Yoneda lemma, limits and colimits, adjunctions, monads — revisits a broad range of mathematical examples from the categorical perspective. 2016 edition.

Category theory emerged in the 1940s in the work of Samuel Eilenberg and Saunders Mac Lane. It describes relationships between mathematical structures. Outside of pure mathematics, category theory is an important tool in physics, computer science, linguistics, and a quickly-growing list of other sciences. This book is about 2-dimensional categories, which add an extra dimension of richness and complexity to category theory. 2-Dimensional Categories is an introduction to 2-categories and bicategories, assuming only the most elementary aspects of category theory. A review of basic category theory is followed by a systematic discussion of 2-/bicategories, pasting diagrams, lax functors, 2-/bilimits, the Duskin nerve, 2-nerve, internal adjunctions, monads in bicategories, 2-monads, biequivalences, the Bicategorical Yoneda Lemma, and the Coherence Theorem for bicategories. Grothendieck fibrations and the Grothendieck construction are discussed next, followed by tricategories, monoidal bicategories, the Gray tensor product, and double categories. Completely detailed proofs of several fundamental but hard-to-find results are presented for the first time. With exercises and plenty of motivation and explanation, this book is useful for both beginners and experts.

Topos Theory is an important branch of mathematical logic of interest to theoretical computer scientists, logicians and philosophers who study the foundations of mathematics, and to those working in differential geometry and continuum physics. This compendium contains material that was previously available only in specialist journals. This is likely to become the standard reference work for all those interested in the subject.

Category Theory has developed rapidly. This book aims to present those ideas and methods which can now be effectively used by Mathe maticians working in a variety of other fields of Mathematical research. This occurs at several levels. On the first level, categories provide a convenient conceptual language, based on the notions of category, functor, natural transformation, contravariance, and functor category. These notions are presented, with appropriate examples, in Chapters I and II. Next comes the fundamental idea of an adjoint pair of functors. This appears in many substantially equivalent forms: That of universal construction, that of direct and inverse limit, and that of pairs offunctors with a natural isomorphism between corresponding sets of arrows. All these forms, with their interrelations, are examined in Chapters III to V. The slogan is "Adjoint functors arise everywhere". Alternatively, the fundamental notion of category theory is that of a monoid -a set with a binary operation of multiplication which is associative and which has a unit; a category itself can be regarded as a sort of general ized monoid. Chapters VI and VII explore this notion and its generaliza tions. Its close connection to pairs of adjoint functors illuminates the ideas of universal algebra and culminates in Beck's theorem characterizing categories of algebras; on the other hand, categories with a monoidal structure (given by a tensor product) lead inter alia to the study of more convenient categories of topological spaces.

An introduction to the theory of toposes which begins with illustrative examples and goes on to explain the underlying ideas of topology and sheaf theory as well as the general theory of elementary toposes and geometric morphisms and their relation to logic.